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Integrable and non-integrable Lotka-Volterra systems

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ABSTRACT

In a recent paper [1], completely integrable cases were discovered of the Lotka Volterra Hamiltonian (LVH) system without linear terms, $\frac{dx_i}{dt} = \dot{x}_i = \sum_{j=1}^{n} a_{ij}x_ix_j$, $i = 1, \ldots, n$, $a_{ij} = -a_{ji}$, satisfying the condition $H = \sum_{i=1}^{n} x_i = h = const$. In this paper, we first generalize this system to one that includes an arbitrary set of linear terms that preserve the Hamiltonian integral. We thus discover a wide class of LVH systems which we claim to be integrable, since their equations possess the Painlevé property, i.e. their solutions have only poles as movable singularities in the complex t-plane. Next, we focus on the case $n = 3$ and vary some of the parameters, including additional nonlinearities to look for nonintegrable extensions with interesting dynamical properties. Our results suggest that, in this class of systems, non - integrability generally yields simple dynamics far removed from the type of complexity one expects from non - integrable 3 - dimensional nonlinear systems.

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1. Introduction

Integrable and non-integrable Lotka Volterra systems of nonlinear ordinary differential equations (ODEs) have long been a subject of intensive analytical and numerical study. Many integrable infinite and finite-dimensional such systems have been derived through Lax representations, possessing families of first integrals [2]. More recently, in the Hamiltonian case, integrable cases have been constructed using Lie algebraic techniques [3], or by establishing Liouville integrability of the associated ODEs [4].

Lotka-Volterra (LV) systems have a long history due to their implications regarding models of competing species in mathematical biology [19]. In the last 30 years, they have been intensively studied due to many advances in the theory of integrability (see e.g. [7,13,2,9]) and chaotic dynamics (see e.g. [20,21]). Regarding specific applications, many publications have been inspired by physical as well as biological sciences. For example, in [5] the authors generalize the results of previous studies and use bifurcation theory to argue that realistic predator-prey systems are expected to exhibit chaotic dynamics, while in [6], a family of LV systems of nonlinear differential equations that embodies standard models of biological interest is analyzed mathematically. Concerning mathematical aspects of integrable LV equations of relevance to physics the reader is referred to [7–14].

Our main objective here is first to extend our recent results on the integrability of Hamiltonian Lotka-Volterra (LVH) systems [1] to the important case of including linear terms that represent growth and/or loss of populations. Secondly, we have varied the linear coefficients and also introduced nonlinear terms to break integrability and study how “far” these models are from exhibiting physically interesting phenomena one expects from non-integrable dynamical systems.

As is well - known, one widely used method for identifying completely integrable systems of ODEs proceeds via the verification that they possess the Painlevé (P-) property, i.e. the only movable singularities of their solutions in the complex t - plane are poles [15]. This approach has already proved fruitful for identifying integrable LV systems in several studies [16,17]. In fact, it was used extensively in a remarkable paper [18] to explicitly integrate and solve a great number of 3 - dimensional LV systems.

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